MOTION OF TWO-PHASE FLUIDS IN TUBES OF VARIABLE CROSS SECTION WITH LOCAL INPUTS OF MASS AND ENERGY

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One-dimensional transient flows of two-phase fluids are examined which are mixtures of an ideal gas, which does not conduct heat, and solid particles. The problem of the flow of the two-phase mixture into a tube of variable cross section which contains another mixture is solved in the self-similar formulation. Here the mixtures can be both inert and combustible. The problem is reduced to the Cauchy problem for a system of ordinary differential equations with auxiliary conditions at the inner surfaces of the discontinuity. Perturbation theory is used to solve the non-self-similar problem of flow of the combustible gas into the inert two-phase mixture.

1. Let there be a quiescent mixture of gas and solid particles in a tube of variable cross section. The motion of the gas with the particles is examined within the framework of the model of interacting continua. It is assumed that the gas is ideal and does not conduct heat and that the particles are rigid spheres of identical radius. Brownian motion of the particles, their volume fraction, and collisions with each other are not considered. There are viscous and thermal interactions between phases.

At time $t = 0$ a two-phase mixture with other particles and gas starts to enter the tube through an inlet orifice of negligibly small dimensions. As a result, transient motion, which is assumed one-dimensional, arises in both mixtures. The cross-sectional area of the tube varies in the following manner: $F(r) = br^{\nu-1}$, $1 \le \nu \le 3$ (r is the distance from the inlet orifice, ν and b are constants, and $\nu = 1$, 2, or 3 for plane, cylindrical, and spherical geometry.

The flow of mass and heat through the inlet is approximated by power-law functions

$$M_i(t) = bm_i t^{\alpha},\ N(t) = bnt^{\beta}, \tag{1.1}$$

where m_i, n, α and β are constants and $i = 1, 2$. Hereafter, unless stated otherwise, the index 1 refers to the gas and 2 to the particles. The magnitudes (1.1) are related to a unit length and area in cylindrical and plane cases, respectively.

We will assume an initial phase-density distribution of the unperturbed fluid in the tube to be $\rho_{i0} = A_i r^{-\omega}$, where A_i and ω are constants. The analogous problem has been solved [1] in the absence of particles for $\omega = 0$. The equations of motion for the mixture have the form [2]

$$\begin{gathered}\frac{\partial\rho_i}{\partial t} + \frac{\partial\rho_i v_i}{\partial r} + \frac{\nu-1}{r}\rho_i v_i = 0,\ \ \rho_i\frac{d_i v_i}{dt} = (i-2)\frac{\partial p}{\partial r} + (-1)^i f,\\ \rho_i\frac{d_i e_i}{dt} = (-1)^i q + (2-i)\left[f(v_1 - v_2) - p\left[\frac{\partial v_i}{\partial r} + (\nu-1)\frac{v_i}{r}\right]\right],\\ \frac{d_i}{dt} = \frac{\partial}{\partial t} + v_i\frac{\partial}{\partial r},\ e_1 = c_V T_1,\ e_2 = cT_2,\ p = (\gamma-1)\rho_1 e_1,\end{gathered} \tag{1.2}$$

where ρ_i, v_i, e_i, and T_i are the density, velocity, internal energy, and temperature of the phase; p is the pressure; c_V and c are the heat capacities; $f = H\rho_2 e_1^k (v_1 - v_2)$ is the interphase interaction force; $q = \sigma_1 H\rho_2 e_1^k (e_1 - e_2 c_V/c$ is the rate of thermal interaction; H and σ_1 are characteristic constants of the interaction, where σ_1 is dimensionless, and H has dimensions $L^{-2k}T^{2k-1}$. The particles are assumed to be chemically inert.

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The problem is self-similar [3], if at $\omega \neq 0$ the initial phase energies can be neglected ($e_{i0} = 0$) and the following conditions are satisfied:

$$(1+\alpha)(2+\nu-\omega) = (\nu-\omega)(3+\beta),$$
$$k = 0{,}5(1-\delta)^{-1},\ \delta = 0{,}5(2+\beta-\alpha). \tag{1.3}$$

If Eqs. (1.3) are fulfilled and $e_{i0} \neq 0$, only the equilibrium flows, investigated in [4], are self-similar.

We introduce dimensionless functions:

$$V_i = v_i/v_{1s},\ R_i = \rho_i/\rho_{i0},\ \varepsilon_i = T_i/T_{1s}, \tag{1.4}$$

where the index s indicates the gas parameters in the shock wave. These functions depend only on the dimensionless parameter $\lambda = r/r_s$, where $r_s(t)$ is the coordinate of the front of the shock wave which propagates through the mixture in the tube. Dimensional analysis [3] yields $r_s(t) = \sqrt{n/m_1 t^{\delta}/\lambda_0}$. Here λ_0 is a constant to be determined. The particle parameters are considered constant in the shock wave, but the gas parameters are determined from the Rankine–Hugoniot equations.

In the dimensionless functions of (1.4), Eq. (1.2) takes the form

$$R_i' S_i + \left(V_i' + \frac{\nu-1}{\lambda} V_i\right) R_i = 0,$$
$$2R_1S_1V_1' + (\gamma-1)\left[(R_1\varepsilon_1)' - \frac{\omega R_1\varepsilon_1}{\lambda}\right] + GV_1R_1 = -\varkappa BR_2\varepsilon_1^k(V_1-V_2),$$
$$R_1S_1\varepsilon_1' - (\gamma-1)\left[R_1'S_1 - \frac{\omega V_1R_1}{\lambda}\right]\varepsilon_1 + GR_1\varepsilon_1 =$$
$$= \varkappa BR_2\varepsilon_1^k\left[(V_1-V_2) - \frac{1}{2}\sigma_1(\varepsilon_1-\varepsilon_2)\right], \tag{1.5}$$
$$2S_2V_2' + GV_2 = B\varepsilon_1^k(V_1-V_2),\ S_2\varepsilon_2' + G\varepsilon_2 = \frac{1}{2}\sigma_2B\varepsilon_1^k(\varepsilon_1-\varepsilon_2),$$
$$\text{where } S_i = V_i - \frac{\gamma+1}{2}\lambda \quad (i=1,\,2);\ G = \frac{\delta-1}{\delta}(\gamma+1);$$
$$\varkappa = A_2/A_1;\ \sigma_2 = \frac{c_V}{c}\sigma_1;$$
$$B = (\gamma+1)H\left[\frac{2}{(\gamma+1)^2}\right]^k \delta^{\delta/(1-\delta)}\left(\sqrt{\frac{n}{m_1}}\Big/\lambda_0\right)^{1/(1-\delta)};$$

the primes denote differentiation with respect to λ.

In the shock wave ($\lambda = 1$), the parameters of the fluid are described in the form

$$V_1(1) = \varepsilon_1(1) = 1,\ R_1(1) = (\gamma+1)/(\gamma-1),$$
$$V_2(1) = \varepsilon_2(1) = 0,\ R_2(1) = 1. \tag{1.6}$$

Thus, when a shock wave arises in the fluid, the problem reduces to the Cauchy problem for ordinary differential equations (1.5) with initial conditions (1.6) on the right end of the integration path [0, 1] and with auxiliary conditions on the inner surfaces of the discontinuity.

Due to the high dimensionality and strong nonlinearity of the equations, a complete qualitative investigation of the problem is difficult. However, special sets of the system (1.5) can be found which are related to the discontinuities and their properties can be found:

a) The set $V_1 = 0.5(\gamma+1)\lambda$,

$$\varepsilon_1^k B\lambda R_2[\sigma_1(\varepsilon_2-\varepsilon_1) + 2(V_1-V_2)^2] = 2\varepsilon_1R_1[\lambda G + (\gamma-1)\omega]$$

corresponds to the contact discontinuity with respect to the gas phase; it is realized for $\nu > \omega$, $\delta < 2/(2 + \gamma\omega - \omega)$.

b) The set $V_2 = 0.5(\gamma + 1)\lambda$ corresponds to the contact discontinuity with respect to the particles. In order that the density of the solid phase ρ_2 be finite at the discontinuity the condition $\varepsilon_1{}^k B(V_1 - V_2) = GV_2$ must be fulfilled at the discontinuity, which occurs in calculations using the method of characteristics.

c) The surface $2\left(V_1 - \frac{\gamma+1}{2}\lambda\right)^2 = \gamma(\gamma-1)\varepsilon_1$ is a surface of weak discontinuities in the gas: when substituted into the Rankine–Hugoniot conditions it maps onto itself.

When the solution intersects with the surface $(\gamma-1)^2\varepsilon_1 = 4\left(V_1 - \frac{\gamma+1}{2}\lambda\right)$ there has to be a shock at the surface $\varepsilon_1 = 0$. The necessity for this has been derived [1].

For $\varepsilon_1 = 0$, Eq. (1) can be integrated easily

$$c_i V_i^\delta = \left(\delta V_i - \frac{\gamma+1}{2}\lambda\right)^\delta,\ d_i R_i\left(V_i - \frac{\gamma+1}{2}\lambda\right)\lambda^{\nu-\omega-1} = V_i^{\delta(\nu-\omega)/(\delta-1)},\quad \varepsilon_2 = \text{const}\, V_2^2.$$

In order to determine the jump in gas density at the contact discontinuity along the gas, we use a mass and energy balance to determine the constant λ_0 and the jump in internal particle energy at the contact discontinuity along the particles:

$$\int_0^{\lambda_i} R_i\lambda^{\nu-\omega-1}\,d\lambda = \frac{m_i}{A_i(\alpha+1)}\left(\lambda_0\sqrt{m_1/n}\right)^{\nu-\omega}\quad (i = 1, 2),$$

$$\frac{2\delta^2}{(\gamma+1)^2}\int_0^1\left\{R_1\left(V_1^2+\varepsilon_1\right) + \varkappa R_2\left(V_2^2 + \frac{\sigma_1}{\sigma_2}\varepsilon_2\right)\right\}\lambda^{\nu-\omega-1}\,d\lambda = \frac{n}{A_1(\beta+1)}\left(\lambda_0\sqrt{\frac{m_1}{n}}\right)^{\nu-\omega+2},$$

where λ_i is the coordinate of the contact discontinuity of the i-th phase.

The solution to the problem was investigated numerically. The calculation was done for absolutely identical mixtures with the parameters $\gamma = 1.4$, $\omega = 2$, and $\delta = 0.7$. The results are shown in Fig. 1. The solution contains two contact surfaces and two shock waves. As with the explosion problem [2], a peak in the particle density (the ρ-layer) arises in the perturbed region.

2. Now let the mixture entering the tube be a fuel (such a situation occurs in some internal combustion engines). Then a detonation wave and a flame front can propagate through it. We neglect the chemical reaction kinetics and the thickness of the zone in which it occurs. The equilibrium flows are now self-similar ($V_1 = V_2 = V$ and $e_1 = e_2 = e$). In this case the two-phase fluid can be considered to be a gas with the reduced parameters ρ_* and γ_* [3]. For $\omega \neq 0$, the self-similarity conditions are written in the form $p_0 = 0$, $\alpha = \beta = \nu - \omega - 1$, and $\delta = 1$. The problem, with reversed data (an inert gas entering a region with a combustible mixture) has been examined [5].

We introduce new functions z and W:

$$W(\lambda) = vt/r,\ z(\lambda) = \gamma pt^2/\rho r^2.$$

As has been shown [3], the actual solution to the problem consists of investigating the integral of the equation

$$dz/dW = \Phi(z, W, \gamma, \nu, \omega) \tag{2.1}$$

and analyzing the possible discontinuities. The right side of (2.1) is a known function of its arguments [3]. Because the motion of the inert gas for $W < 1$ is similar to one studied previously [1, 4], it is sufficient to study the motion of the combustible mixture along the other side of the contact discontinuity for $W > 1$.

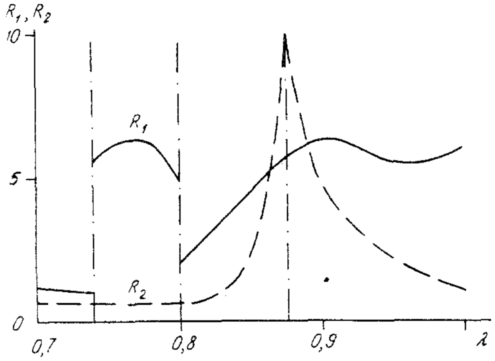

Fig. 1

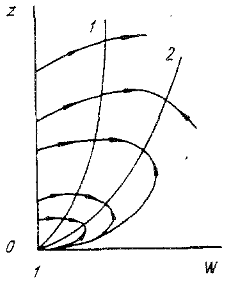

Fig. 2

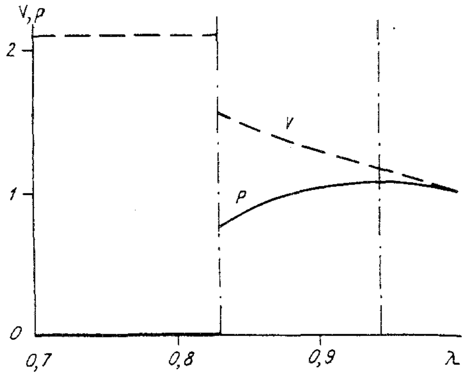

Fig. 3

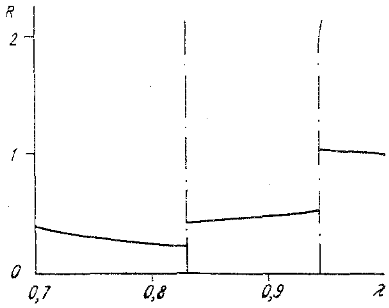

Fig. 4

Let $\omega = 0$. Figure 2 shows the pattern of the field of the integrated curves of Eq. (2.1) for $\nu = 3$ and $W > 1$. The arrows show the direction of decreasing λ; the curves 1 and 2 are for $z = \frac{2\gamma}{\gamma - 1}(W-1)^2$ and $z = (W-1)^2$.

The conditions at the shock with heat evolution can be written in the form

$$\begin{aligned} R_2 &= R_2\left[\frac{\gamma_2}{\gamma_2+1}(1-\Lambda)\left(1+\frac{z_1}{\gamma_1(W_1-1)^2}\right)\right]^{-1}, \\ W_2 &= 1+\frac{\gamma_2}{\gamma_2+1}(1-\Lambda)\left[1+\frac{z_1}{\gamma_1(W_1-1)^2}\right](W_1-1), \\ z_2 &= \left[\frac{\gamma_2}{\gamma_2+1}\right](1-W_1)^2(1-\Lambda)(1+\gamma_2\Lambda)\left[1+\frac{z_1}{\gamma_1(W_1-1)^2}\right]^2, \\ \Lambda^2 &= 1-\frac{\gamma_2^2-1}{\gamma_2^2}\left[1+\frac{2}{(W_1-1)^2}\left[\frac{z_1}{\gamma_1-1}+\frac{Q}{D^2}\right]\right]\left[1+\frac{z_1}{\gamma_1(W_1-1)^2}\right]^{-2}, \end{aligned} \tag{2.2}$$

where the indices 1 and 2 correspond to the combustible mixture and the reaction products; Q is the heat of combustion per unit mass of gas; and D is the velocity of the shock with heat emission.

The inlet corresponds to the point $z = 0$ and $W = \infty$, which can be reached only along the line $z = 0$. The jump at the line $z = 0$, which corresponds to the detonation wave, must be made at the intersection of the integral curve with the parabola

$$z = \frac{1+\gamma_2\Lambda}{1-\Lambda}(W-1)^2. \tag{2.3}$$

For $\Lambda > 0$, the parabola (2.3) is higher than the acoustic parabola

$$z = (W - 1)^2 \tag{2.4}$$

and the solution is unique. For $\Lambda = 0$, the Chapman–Jouguet condition is satisfied, (2.3) coincides with (2.4), and the parabola (2.4) can be reached either from the region $z > (1 - W)$ or from

$$0 < z < (W - 1)^2. \tag{2.5}$$

In the second case, in order to fall in the region (2.5), a simple compression shock must be introduced at the intersection of the integral curve with the parabola $z = a(W - 1)^2$, where $1 < a < 2\gamma_2/(\gamma_2 - 1)$. If $\Lambda < 0$, then (2.3) lies below (2.4), and intersection with the parabola is possible only if an additional density shock is introduced. The position of the additional shock can be determined if the inflow of the mixture into the tube is fixed.

Now let a flame front propagate through the combustible mixture. Because the front is a rarefaction shock, it should have a shock wave, which must be constructed from the intersection point of the integral curve with the parabola $z = \frac{2\gamma_1}{\gamma_1 - 1}(W - 1)^2$, in order to cross over to the line $z = 0$.

The flame front can be constructed from the intersection points of the integral curve with the parabola $z = g(W - 1)^2$. The case $g > 1$ corresponds to subsonic and $g < 1$ to supersonic combustion. For $g = 1$, the Chapman–Jouguet condition is fulfilled for combustion and a simple shock at the flame front is possible. For $g < 1$, the solution exists only with an additional shock.

The outer side of the flame front must correspond to points lying in the region $z_1 > 2\gamma_1(W_1 - 1)^2/(\gamma_1 - 1)$, with $W > 1$. From the conditions (2.2) we obtain

$$\frac{2g(2\gamma_2 - 1)}{\gamma_2(\gamma_2 - 1)} - \frac{g^2}{\gamma_2^2} < \frac{2Q}{u^2} < 1 + \frac{2g}{\gamma_2 - 1}, \tag{2.6}$$

where u is the velocity of the flame front through the combustion products. It follows from (2.6), in particular, that $g \neq \gamma_2$.

For $2Q/u^2 > (2\gamma_2 - 1)^2/(\gamma_2 - 1)^2$, the left side of the inequality in (2.6) is satisfied for any g. If $2Q/u^2 \leq (2\gamma_2 - 1)^2/(\gamma_2 - 1)^2$, then for values of g which satisfy the equation $\frac{2g(2\gamma_2 - 1)}{\gamma_2(\gamma_2 - 1)} - \frac{g^2}{\gamma_2^2} = \frac{2Q}{u^2}$, the flame front merges with the accompanying simple shock into a detonation wave.

Numerical solutions were done for $\omega = 0$, $\nu = 3$, and $p_0 = 0$. The results for the Chapman–Jouguet detonation are shown in Figs. 3 and 4. Parameters are ratioed to the values in the shock wave, which propagates through the inert gas. The equilibrium flows for $\omega = 0$ can also be studied for a nonzero volume fraction of particles. Here, however, the equation of the state of the mixture must be changed: $e = p(A + B\rho)/\rho$, where A and B are constants.

The results can be extended to a nonself-similar problem of the flow of a combustible gas into an inert two-phase mixture, where the interaction terms f and q are computed from more exact formulas

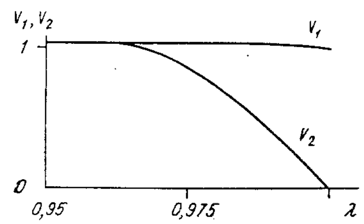

Fig. 5

$$f = (3/8)C_D(v_1 - v_2)|v_1 - v_2|\rho_1\rho_2/(\rho_{22}a),$$
$$q = (3/2)\,\mathrm{Nu}\,k(T_1 - T_2)/(\rho_{22}a^2),$$

where C_D is the drag coefficient; Nu is the Nusselt number; k is the thermal conductivity of the gas; a is the particle radius; and ρ_{22} is the density of the particle material.

Figure 5 shows the parameters of the two-phase mixture to the right of the contact discontinuity, which is found by perturbation theory [6] for $\nu = 1$ and $2\rho_{22}a^2/(9\mu) = 0.01$, where μ is the gas viscosity.

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MODELING TRANSIENT TURBULENT AXISYMMETRIC FLOW IN NARROW GAPS BETWEEN CONTOURED ROTATING SURFACES

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Rotating channels of various shapes are widely used in modern power plants, turbines, and chemical manufacturing equipment. In particular, efficient air-centrifuge classifiers, which are used in the powder technology, and the express analyzers which are based on them [1, 2] make it possible to fractionate powdered materials by particle dimension at a high rate and to determine their granulometric composition. The working zone of these devices is a narrow gap between rotating contoured surfaces, in which the parts flow and are separated by size by resistive and centrifugal forces. A diagram of the separation zone is presented in Fig. 1.

Here the torsional turbulent flow of an incompressible gas is studied, based on the parabolic equations, obtained from the "narrow channel" approximation. The unsymmetric channel is studied when one of the limiting surfaces is flat and perpendicular to the axis of rotation, and the second is contoured such that the gap width varies according to $H = H(R)$, where R is the radius. The transient nature of the flow is caused by the forced change of the rotation rate of the walls Ω or the flow Q through the gap. The problem has been examined in the steady-state formulation [3] based on the two-parameter Launder–Jones model [4, 5].

The operational efficiency of these devices can be further enhanced by establishing their fundamental physical characteristics, which are based on models which adequately describe the hydrodynamics of transient torsional axisymmetric flows which are directed both toward the axis of rotation ($Q < 0$, Fig. 1) and toward the periphery ($Q > 0$).

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